

# EVALUATING COMPLEX SYSTEMS WHEN NUMERICAL INFORMATION IS SPARSE

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## ABSTRACT

Analyzing complex systems for which there is insufficient information for a complete quantitative characterization is a common problem encountered in military and research applications. As a result of repeated experience with this situation, we developed an approach that uses integrated logic modeling and approximate reasoning to make sophisticated and complicated predictions and decisions about systems with significant gaps in quantitative understanding. We describe how a process tree can be used to gain better understanding of complex physical or operational processes. We show how this understanding can be used to develop an approximate reasoning decision model that efficiently uses experience and expert judgment to make reasonable decisions.

**KEYWORDS:** System Modeling, Uncertainty, Approximate Reasoning, Fuzzy Sets.

## INTRODUCTION

Which facilities and equipment in an aging research complex should be replaced first?<sup>1</sup> How can one predict the risk of losing classified information during the visit of a foreign delegation to a secure facility?<sup>2</sup> What is the order in which spare parts should be purchased for a custom-built research system—but provide your answer without an extensive effort in collecting quantitative failure or repair data?<sup>3</sup> Should a material in a high-reliability system be replaced given limited surveillance data and incomplete understanding of the aging of the material in this application?<sup>4</sup> On the surface, these problems seem unrelated, but they have many common characteristics. In each case, the problem centers on a system state that can be reached by many different paths. One can approach this problem by evaluating the relative importance of these different paths in determining the occurrence of the system state of interest. To follow this approach, the analyst lists the possible paths to the state as completely as is feasible and then evaluates each state according to some appropriate figure of merit (FOM). This task is complicated when understanding of the system is insufficient for a completely deterministic or even probabilistic analysis. However, even when quantitative understanding is limited, a great deal is still known about the system and its behavior, but critical segments of the knowledge base are qualitative or intuitive in nature and are held by disparate groups of specialists.

We have encountered problems of this nature so often that we have developed a standard process to address them that we call the Logic Evolved Decision approach (LED).<sup>5</sup> As shown in Fig. 1, we first model the system behavior of interest with a deductive logic model called a system process tree, which gives us an organized list of possible paths leading to the final system state of interest. Sometimes this alone is sufficient to provide a list of factors that combine to provide the appropriate FOM for evaluating the different possibilities. In some very complex cases, we develop a logic model that describes the FOM evaluation process itself. In either situation, we develop a forward-chaining implication structure that combines individual factors using approximate reasoning (AR) techniques<sup>6,7</sup> to produce an FOM evaluation of each of the possible paths to a system state. This inter-linking of logic models to identify possibilities, develop an appropriate FOM, and provide an algorithm for the FOM evaluation of the possibilities is the LED process.

## SYSTEM PROCESS TREE

The system process tree describes how a system state of interest can be reached from possible initial states.<sup>5</sup> A process tree is a visual representation of a set of inter-linked logical equations that model the transitions between states of a system. The inter-connections between logic equations are described by logic gates. A process tree can either model an initial state leading to other final states or how a final state could have resulted from possible initial states. In either case, deductive reasoning is used to establish the links between states. We call the first case “forward deduction” and the latter case “backward deduction,” and the resulting process trees are termed “consequence” and “causal.” In this paper, we concentrate on the causal process tree.

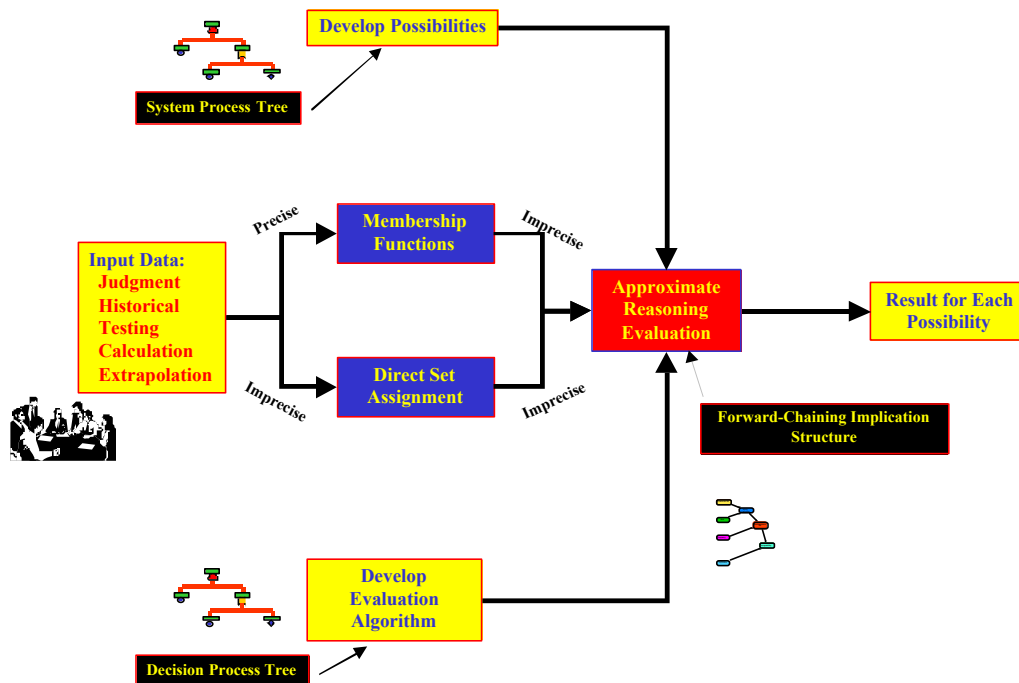


Fig. 1. How the LED Approach Works.

The causal process tree is a structured way to determine what the possible causes of a particular system state could be. Typically, there are many intermediate steps between the final state of interest and the initial states that could lead to it. The intermediate steps can occur because of internal system processes set in motion by changing system conditions or can be driven by external influences on the system. A chain of steps leading from an initial to a final state is called a path. The paths are found by successive substitution in the logic equations that define the process tree.

Process trees can be developed rapidly using a custom-built software called LED Tree. LED Tree provides a visual way to construct the logic equations that describe the process. In fact, the user does not even need to interact with the equation set at all. This tool gives the user much more sophisticated capabilities than can be found in a fault-tree graphical interface. It allows the user to rapidly construct complex process trees using complicated gates, allows logic structures to be repeated in new contexts, and allows the use of parts of sentences in the description field of the process tree. With care, paths through the tree are natural language descriptions of the process they describe. LED Tree output can be translated into digraph and other logical representations of the process tree, giving the user great flexibility in visualizing and analyzing problems.

## **APPROXIMATE REASONING EVALUATION OF POSSIBILITIES**

In our FOM evaluations, we are often forced to include significant amounts of qualitative knowledge. We do this through the techniques of AR, which provides a mathematically structured way of using imprecise knowledge. We have found it useful to draw a distinction between precision and uncertainty. A parameter may take on values between 1.02 and 3.0 following a truncated normal distribution. Although the parameter value is uncertain, its characterization is very precise. On the other hand, a parameter may be described linguistically as being “medium to large.” This is an imprecise and an uncertain measure. We call a parameter that is precisely defined using numerical values a “numerical variable.” A variable that is imprecisely defined using linguistic descriptors is called a “linguistic variable.” A numerical variable takes on numerical values, whereas a linguistic variable takes on linguistic values. Uncertainty about a numerical value can be described using probability distributions. Uncertainty about a linguistic value can be expressed using fuzzy measures.

We believe that when data with different levels of precision are combined, the more precise information should be reduced to the same precision as the less precise information. Thus, when numerical and linguistic variables are combined, the numerical information is reduced to linguistics using membership functions.<sup>6</sup> The linguistic values used in our evaluations are typically elicited from knowledgeable workers in the field of interest. We call what we elicit from the experts “conviction.” We have used the term “conviction” to distinguish our measure from the terms “belief” and “possibility” that are already used in the literature with specific meanings. “Conviction” implies that we have allowed the expert to express a number of different aspects of his uncertainty using a single fuzzy measure. Conviction includes the informant’s uncertainty about which linguistic variable is appropriate, allows them to “hedge” between linguistic values,

captures the ambiguity inherent in linguistic variables, and expresses variability in the subject matter. We have found that experts find this a natural thing to do and, in many circumstances we have encountered, prefer using linguistics with conviction expressions to making probabilistic statements.

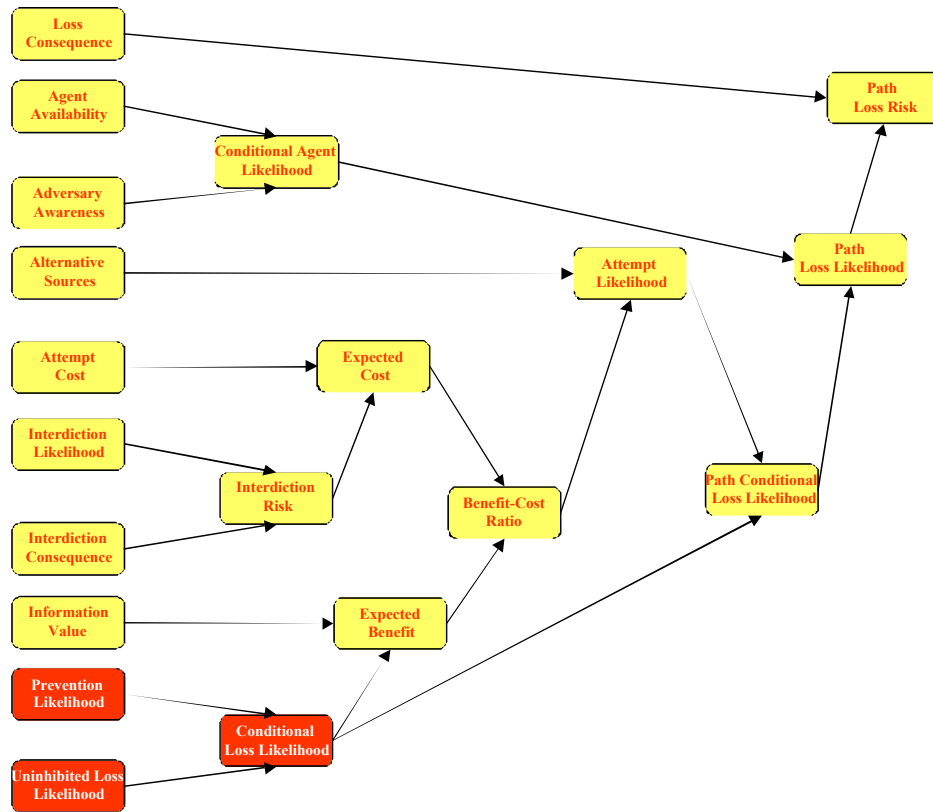
Conviction values follow a slightly different set of axioms than possibilities in that the conviction of the universe is 1.0, but no proper subset of the universe need have a value of 1.0. Otherwise, conviction obeys the mathematics of possibility and is just one of many manifestations of a possibility measure.<sup>6</sup> What we are calling “conviction” is an inherently imprecise measure. There is no uniquely reproducible basis, such as a long-run frequency, as a basis for probability or even a wager paradigm. The use of numbers is a convenient vehicle for expressing and propagating imprecise convictions about a proposition.\*

The method for evaluating the possibilities can be described as a forward-linked implication structure. An example is shown in Fig. 2 for evaluating the classified information compromise risk posed by foreign visitors to a secure facility. The terminal nodes along the left side of the diagram are input data. Each node where two or more inputs come together represents a rule set that combines two or more linguistic variables to produce an output linguistic variable. An example of a rule set is shown in Fig. 3. This rule is used to combine the Uninhibited Loss Likelihood and Prevention Likelihood variables to produce Conditional Loss Likelihood values as shown by the red nodes in Fig. 2. In the example, Uninhibited Loss Likelihood is modified by Prevention Likelihood to produce Conditional Loss Likelihood. If the Uninhibited Loss Likelihood value (shown across the top of the table) is Likely and the Prevention Likelihood value (shown along the left edge of the table) is Unlikely, the output Conditional Loss Likelihood is read from the table as Likely. Conviction values are propagated using a Min-Max process. The value of conviction for any entry in the rule set is the minimum of the conviction for the inputs. The conviction for the output value when it results from more than one input set is the maximum of the conviction values for that output value. A careful study of Fig. 3 will make this operation clear. The output of each rule base provides input to other rule bases. The final result is the output of the terminal node on the right of Fig. 2. We can develop complicated implication structures using a formal logic diagram approach treating this FOM evaluation as a process. The resulting decision process tree identifies the factors taken into account and how they are combined to arrive at a final result. The decision process tree can be converted automatically into a linked rule set that can then be used in an automated evaluation of many paths in a short time.

The process tree for a complex system may yield a large number of paths. In addition, the FOM evaluation implication structure may be quite complex with many variables and rules. The evaluation of these possibilities can be very time consuming. We have developed an algorithm called “Fast Min Max” (FMM) that greatly speeds up this

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\*We could just as well have used linguistic descriptors, but then we would also have to supply a set of rules for propagating the values through implication chains. Such a proliferation of parallel rule sets would quickly become cumbersome.



**Fig. 2. A Forward-Linked Implication Structure.**

Unlikely and Likely Produces Likely

Uninhibited Loss Likelihood					
	Very Unlikely (0)	Unlikely (0)	Likely (3)	Nearly Certain (7)	
Very Unlikely (0)	Very Unlikely (0)	Unlikely	Likely	Nearly Certain	
Unlikely (5)	Very Unlikely (0)	Unlikely	Likely $\text{Min}(.3, .5) \rightarrow .3$	Likely $\text{Min}(.7, .5) \rightarrow .5$	
Likely (.5)	Very Unlikely (0)	Unlikely	Unlikely $\text{Min}(.3, .5) \rightarrow .3$	Unlikely $\text{Min}(.7, .5) \rightarrow .5$	
Nearly Certain (0)	Very Unlikely (0)	Very Unlikely (0)	Very Unlikely (0)	Very Unlikely (0)	

Maximum of Minima for Likely Produces Membership of .5 in Likely

**Fig. 3. An Implication Rule Base.**

process. In this algorithm, the linked rule bases are reduced to a set of implication equations containing only the input data. The Min-Max principle is applied to this implication equation set for each path. Tests have indicated that this method is efficient way to provide FOM evaluations for large numbers of paths.

One of the challenges of using AR approaches is the interpretation of the results. The most accurate picture of the situation is achieved when the possibility distribution of the result is shown. This shows the spread in conviction among the possible values of the result and captures most accurately the view of the experts or data used in the analysis. However, this result can leave people confused. Often they would rather see a “statistic” of the distribution, analogous to a mean or some percentile of a statistical distribution. The process of generating such a statistic is called defuzzification. There are many methods for generating statistics. One popular method using a centroid produces a result analogous to a mean.<sup>9</sup> Using this method, a possibility distribution across several values of the output linguistic variable is reduced to a single linguistic value that captures where the bulk of the conviction lies.

## CONCLUSIONS

The approach outlined above is useful for addressing questions about complex systems when precise information is insufficient for a complete analysis. It is also useful for rapid, approximate evaluations when more precise data would be difficult or time consuming to develop. We try to adapt the precision of our evaluation to the true precision of the input knowledge and avoid creating greater precision than is supported by the knowledge base. We have successfully evaluated some very complex systems with very sparse numerical information bases, but with significant qualitative understanding spread among a group of experts. We have found that for many systems, AR and linguistic variables are a natural way for the experts to express their state of knowledge and beliefs.

## REFERENCES

1. T. F. Bott and S. W. Eisenhower, “**A Logic Model Approach to Conceptual Design of a Scientific/Industrial Complex**,” *Proceedings of the 2002 ASME Pressure Vessel and Piping Conference*, to appear August 2002.
2. Terry F. Bott, “**Evaluating the Risk of Industrial Sabotage**,” *Proceedings of the 1999 Annual Reliability and Maintainability Symposium*, IEEE, Jan 18-21, Washington, D.C.
3. S. W. Eisenhower, T. F. Bott and J W. Jackson, **Prioritizing the Purchase of Spare Parts Using an Approximate Reasoning Model**, 2002 RAMS Proceedings, January 28-31, 2002, Seattle WA.
4. T. F. Bott and S. W. Eisenhower, **An Approach to Assessing the Need for High Explosives Replacement in Aging Nuclear Weapons**, 27<sup>th</sup> International Pyrotechnics Seminar, July 10-14, 2000, Grand Junction CO, LA-UR-00-2206.
5. S. W. Eisenhower and T. F. Bott, **Application of Approximate Reasoning to Safety Analysis**, Los Alamos National Laboratory report LA-UR-99-1932/*Proceedings 14<sup>th</sup> International System Safety Conference: System Safety*, August 1999.
6. Ramon Lopez de Mantaras, *Approximate Reasoning Models*, Ellis Horwood Series in Artificial Intelligence, Ellis Horwood LTD, 1990.
7. D. Dubois and H Prade, *Possibility Theory*, Plenum Press, 1988.
8. T. J. Ross, *Fuzzy Logic with Engineering Applications*, McGraw-Hill, New York, 1995.